Metasurface Synthesis Using Momentum Transformation

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We introduce a novel approach to model spatial discontinuities, such as thin surfaces and metasurfaces, in the momentum space (spatial spectrum). The approach is called "momentum transformation" to emphasize its spatial spectral nature and avoid confusion with temporal spectral techniques.

The momentum transformation approach is analogous to expressing Maxwell's equations in the sense of distributions* but in the momentum space and is applicable to scalar and vector waves. The momentum transformation is particularly suitable for the scalar case due to its simplicity.

The momentum transformation has been applied to metasurface synthesis, where it yields a complete description of the metasurface in the momentum space and provides physical insight into the transformation operation.

These slides are adopted from an oral presentation at URSI GASS 2014, Beijing, China.

Keywords: 3D Metamaterials, Metasurfaces, Frequency Selective Surfaces, Metasurface Synthesis Technique, Momentum Conservation, Comparison with Optics, Momentum Transformation.

Outline

I. Motivation

II. MT Method

III. Comparison with Optics

IV. Examples

❖ Conclusions & Questions

Outline

I. MOTIVATION

II. MT METHOD

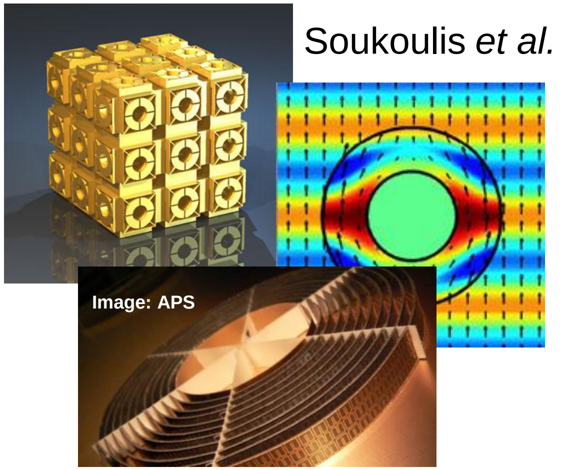
III. COMPARISON WITH OPTICS

IV. EXAMPLES

❖ CONCLUSIONS & QUESTIONS

Why are Metasurfaces Interesting?

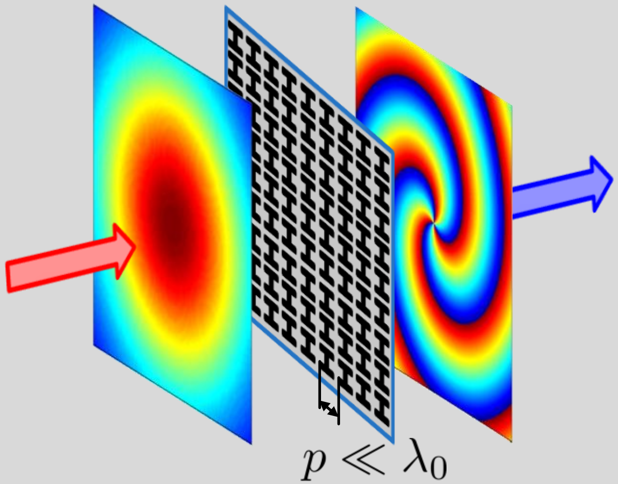
3D Metamaterials



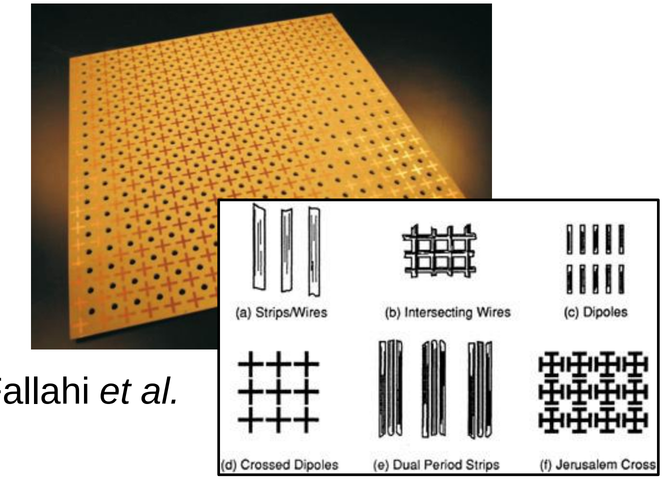
Soukoulis *et al.*

Smith *et al.*

Metasurfaces



Frequency Selective Surfaces (FSS)



Fallahi *et al.*

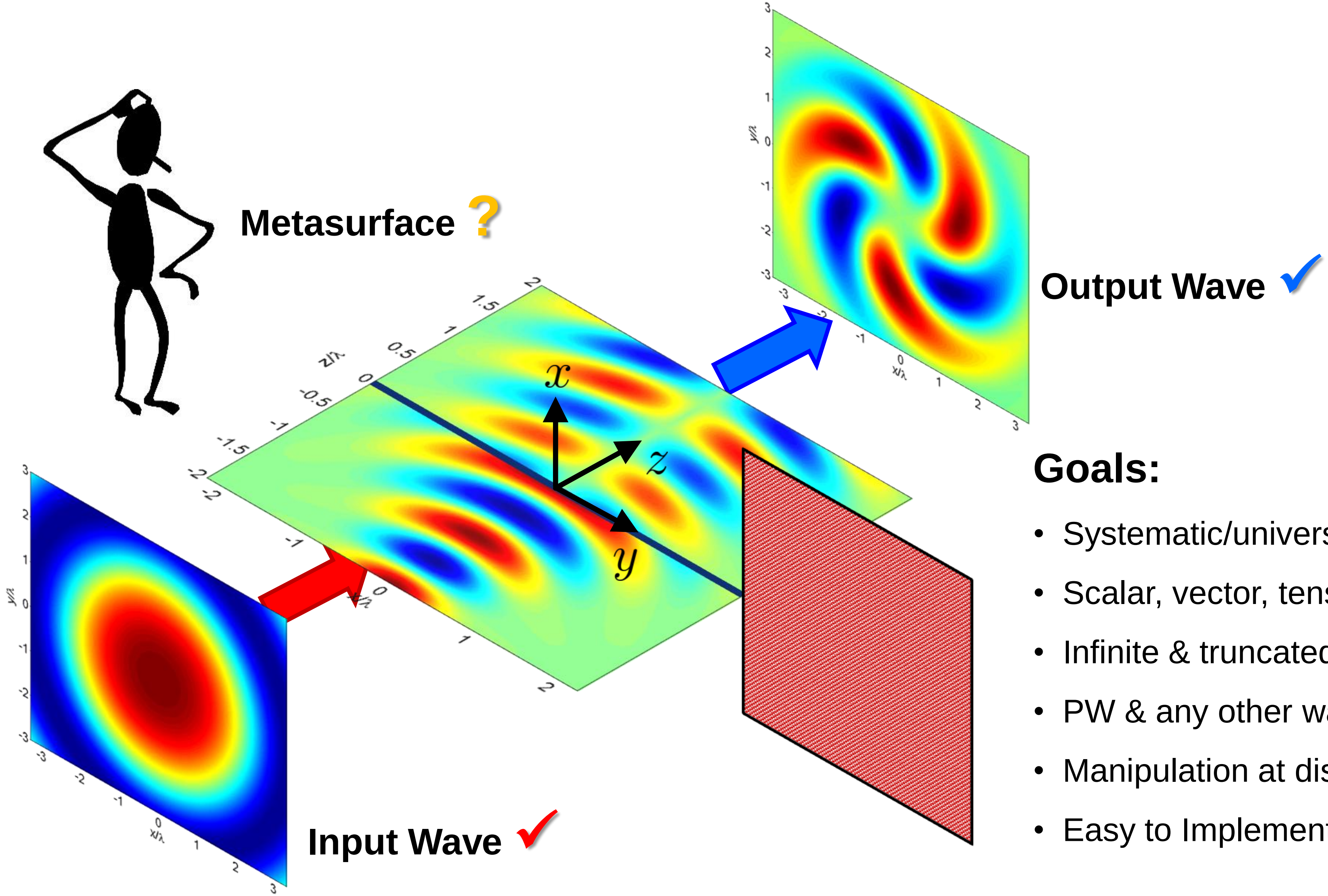
3D Metamaterials
- ☺ Extra Field Control
- ☺ Signal Processing
- ☹ Bulkiness
- ☹ High Loss
- ☹ Fabrication Difficulty

Metasurfaces
- ☺ Extra Field Control
- ☺ Reduced Dimensionality
- ☺ Extensive Processing
- ☺ Low Loss
- ☺ Fabrication Easiness
- ☺ Incidence Flexibility

Frequency Selective Surfaces (FSS)
- ☺ Reduced Dimensionality
- ☺ Filtering & Polarization
- ☹ Limited Signal Processing
- ☹ Uniformity Restriction

Dimensional Reduction of Metamaterials & Functional Extension of FSSs

Metasurface Synthesis Technique ?



Goals:

- Systematic/universal
- Scalar, vector, tensor
- Infinite & truncated MS
- PW & any other wave
- Manipulation at distance
- Easy to Implement

Benefits:

- Physical insight
- Inspiring new devices

SYNTHESIS → TWO-STEPS:

- **Determination of metasurface transfer function**
- **Implementation (scatterring particles)**

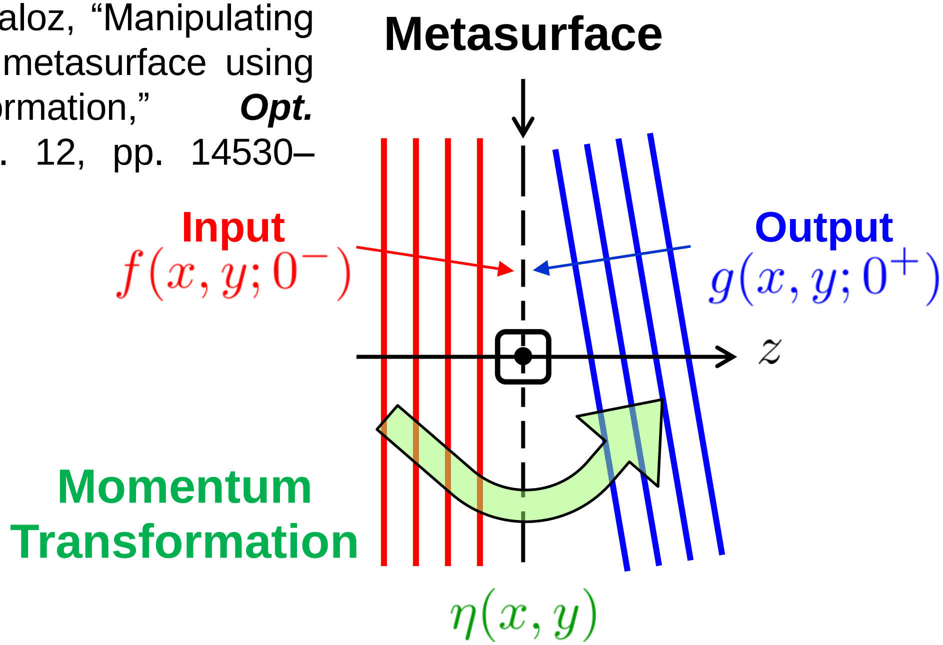
Outline

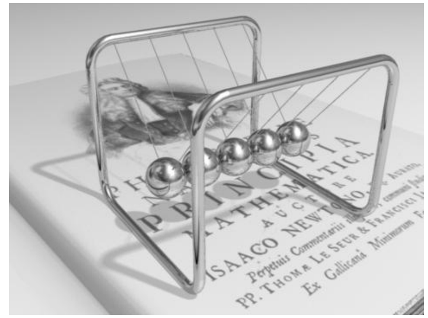
MT*: Momentum Transformation

(k, not “spectral” avoid confusion with ω)

Foundation for the MT Method: Momentum Conservation

M. A. Salem and C. Caloz, “Manipulating light at distance by a metasurface using momentum transformation,” ***Opt. Express***, vol. 22, no. 12, pp. 14530–14543, Jun. 2014.





$$h\left(x',y';x,y\right) \neq h\left(x-x',y-y'\right)$$

Green function

Linear shift-variance: $g\left(x,y\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left(x',y'\right) h\left(x',y';x,y\right) dx'dy'$

Zero EM thickness ⇒ locality: $h\left(x',y';x,y\right) = \delta\left(x'-x,y'-y\right)\eta\left(x',y'\right)$

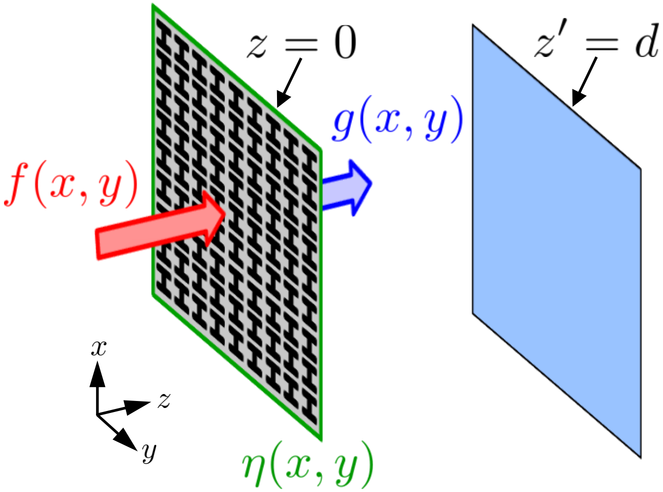
Consequence of locality: $g\left(x,y\right) = f\left(x,y\right)\eta\left(x,y\right)$

Fourier transform: $\tilde{g}\left(k_x,k_y\right) = \tilde{f}\left(k_x,k_y\right) * \tilde{\eta}\left(k_x,k_y\right)$

$\Downarrow$

Momentum shift-invariance ≡ momentum conservation: $\tilde{f}\left(k_x-K_x,k_y-K_y\right) \overset{z=0}{\Rightarrow} \tilde{g}\left(k_x-K_x,k_y-K_y\right)$

MT Method: Scalar Case



Locality: $g(x,y) = \eta(x,y) f(x,y) \rightarrow \eta(x,y) = \dfrac{g(x,y)}{f(x,y)}$

Metasurface: $\eta(x,y) = g(x,y)\xi(x,y), \quad \xi(x,y) = \dfrac{1}{f(x,y)}$

Momentum conservation: $\tilde{\eta}(k_x,k_y) = \tilde{g}(k_x,k_y) * \tilde{\xi}(k_x,k_y)$

Plane wave (Fourier) expansion at z = 0 of $\psi = f, \eta, g$**:**

$$\psi(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{\psi}(k_x,k_y)\, e^{+i(k_x x + k_y y)} dk_x dk_y$$

$$\tilde{\psi}(k_x,k_y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\psi(x,y)\, e^{-i(k_x x + k_y y)} dx dy$$

Momentum wave equation:

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

$$\left(e^{-i\omega t}\right)$$

Specification at z = d: $g(x,y;z'=d) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\underbrace{\tilde{g}(k_x,k_y)\, e^{-ik_z d}}_{\tilde{g}_d(k_x,k_y)} e^{+i(k_x x + k_y y)} dk_x dk_y$

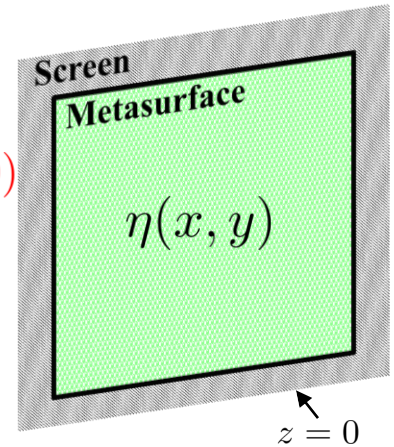
Reverse propagation: $\tilde{g}(k_x,k_y) \rightarrow \tilde{g}_d(k_x,k_y) = \tilde{g}(k_x,k_y)\tilde{\varphi}(k_x,k_y;k;d)$, $\boxed{\tilde{\varphi}(k_x,k_y;k;d) = e^{-i\sqrt{k^2-k_x^2-k_y^2}\,d}}$

Momentum transformation equation: $\boxed{\tilde{\eta}(k_x,k_y) = \tilde{g}(k_x,k_y)\,\tilde{\varphi}(k_x,k_y;k,d) * \tilde{\xi}(k_x,k_y)}$

Momentum Conservation Method: Vector Case

VPW orthogonality: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{E}_{\perp}^{\mathrm{VPW}}(x,y;k_z) \times \mathbf{H}_{\perp}^{*\mathrm{VPW}}(x,y;k_z')\,dxdy = P_{k_z}^{\mathrm{VPW}}\delta(k_z - k_z')$

$\mathbf{E}_{\perp}^{-}(x,y,0) \downarrow \tilde{\zeta}^{-}(k_z)$



$\mathbf{E}_{\perp}^{+}(x,y,0) \downarrow \tilde{\zeta}^{+}(k_z)$

k_z eigenmode expansion:

$$\mathbf{E}^{\mp}(x,y;z) = \int_{0}^{+\infty} \tilde{\zeta}^{\mp}(k_z)\mathbf{E}_{\perp}^{\mathrm{VPW}}(x,y;k_z)\,e^{ik_z z}dk_z$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \ldots \times \mathbf{H}^{*\mathrm{VPW}\mp}(x,y;k_z)dxdy$$

& VPW orthogonality

Scalar expansion coefficients:

$$\tilde{\zeta}^{-}(k_z) = \frac{1}{P_{k_z}^{\mathrm{VPW}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{E}_{\perp}^{-}(x,y) \times \mathbf{H}_{\perp}^{*\mathrm{VPW}}(x,y;k_z)\,dxdy$$

$$\tilde{\zeta}^{+}(k_z) = \frac{1}{P_{k_z}^{\mathrm{VPW}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{E}_{\perp}^{+}(x,y) \times \mathbf{H}_{\perp}^{*\mathrm{VPW}}(x,y;k_z)\,dxdy$$

Reverse propagator: $\tilde{\varphi}(k_x,k_y;k,d) = e^{-ik_z d}$

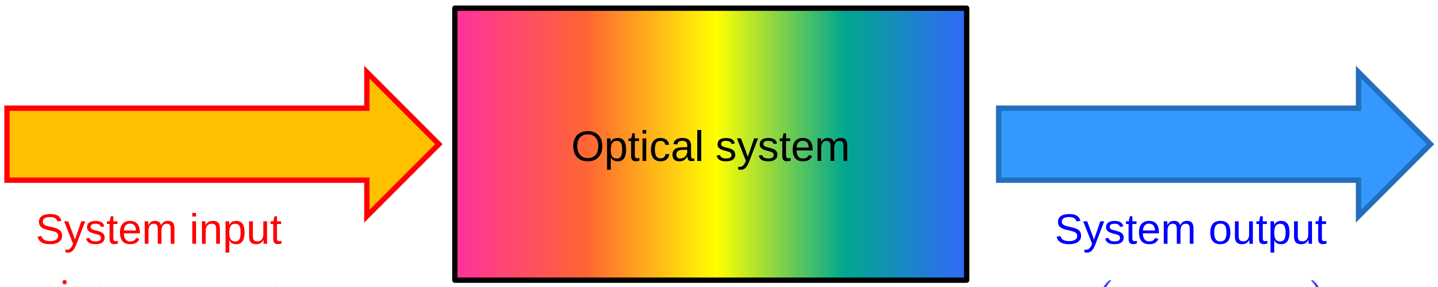
Scalarized (correspondence with scalar case) solution: $\tilde{\zeta}^{-}(k_z) * \tilde{\eta}(k_z) = \tilde{\zeta}^{+}(k_z)\tilde{\varphi}(k_z,d)$

Modal transformation, using $k^2 = k_x^2 + k_y^2 + k_z^2$**:** $\boxed{\tilde{\eta}(k_x,k_y) = \tilde{\eta}(k_z) = \sqrt{k^2 - k_x^2 - k_z^2})}$

Outline

*J. W. Goodman, Introduction to Fourier Optics, 2nd edition, 1996.

The Complex Amplitude Transmittance (CAT)



System input

$f^{\mathrm{i}}(x, y, z_{\mathrm{in}})$

System output

$g(x, y, z_{\mathrm{out}})$

CAT:

$$t(x, y) = \frac{g(x, y, z_{\mathrm{out}})}{f^{\mathrm{i}}(x, y, z_{\mathrm{in}})}$$

Not limited to bulky elements

EXAMPLE 4.1-1. ***Scanning.*** A thin transparency with complex amplitude transmittance $f(x,y) = \exp(j\pi x^2/\lambda f)$ introduces a phase shift $2\pi\phi(x,y)$ where $\phi(x,y) = -x^2/2\lambda f$, so that the wave is deflected at the position (x,y) by the angles $\theta_x = \sin^{-1}(\lambda\partial\phi/\partial x) = \sin^{-1}(-x/f)$ and $\theta_y = 0$. If $|x/f| \ll 1$, $\theta_x \approx -x/f$ and the deflection angle θ_x is directly proportional to the transverse distance x. This transparency may be used to deflect a narrow beam of light. If the transparency is moved at a uniform speed, the beam is deflected by a linearly increasing angle as illustrated in Fig. 4.1-6.

Spatial Light Modulators (SLM)

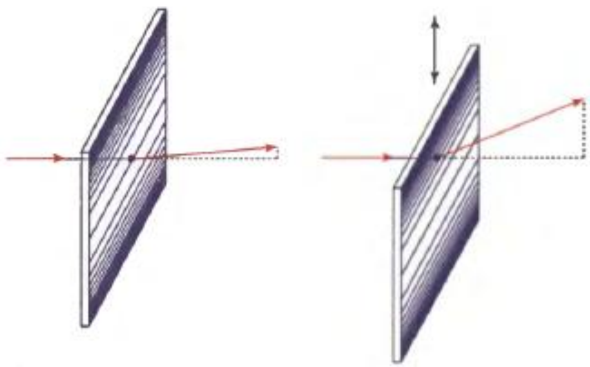


Figure 4.1-6 Using a frequency-modulated transparency to scan an optical beam.

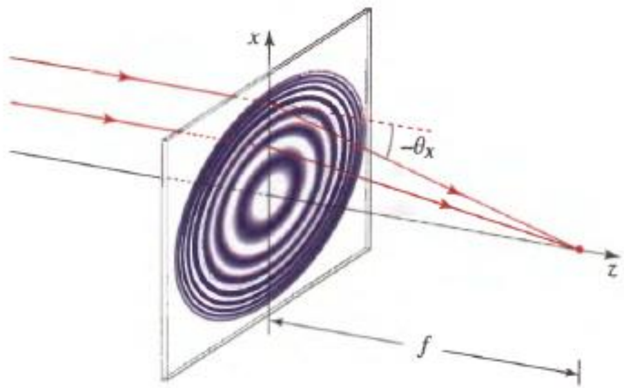
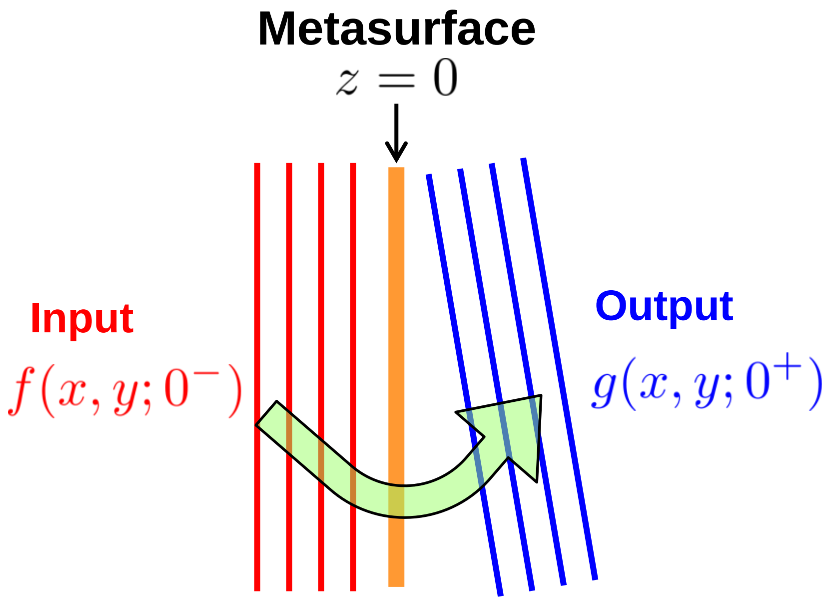


Figure 4.1-7 A transparency with transmittance $f(x,y) = \exp(j\pi x^2/\lambda f)$ bends the wave at position x by an angle $\theta_x \approx -x/f$ so that it acts as a cylindrical lens with focal length f.

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2007.

Transformation in MT and CAT



Momentum transformation (MT):

$$\eta\left(x, y\right) = \frac{g\left(x, y, 0^+\right)}{f\left(x, y, 0^-\right)}$$

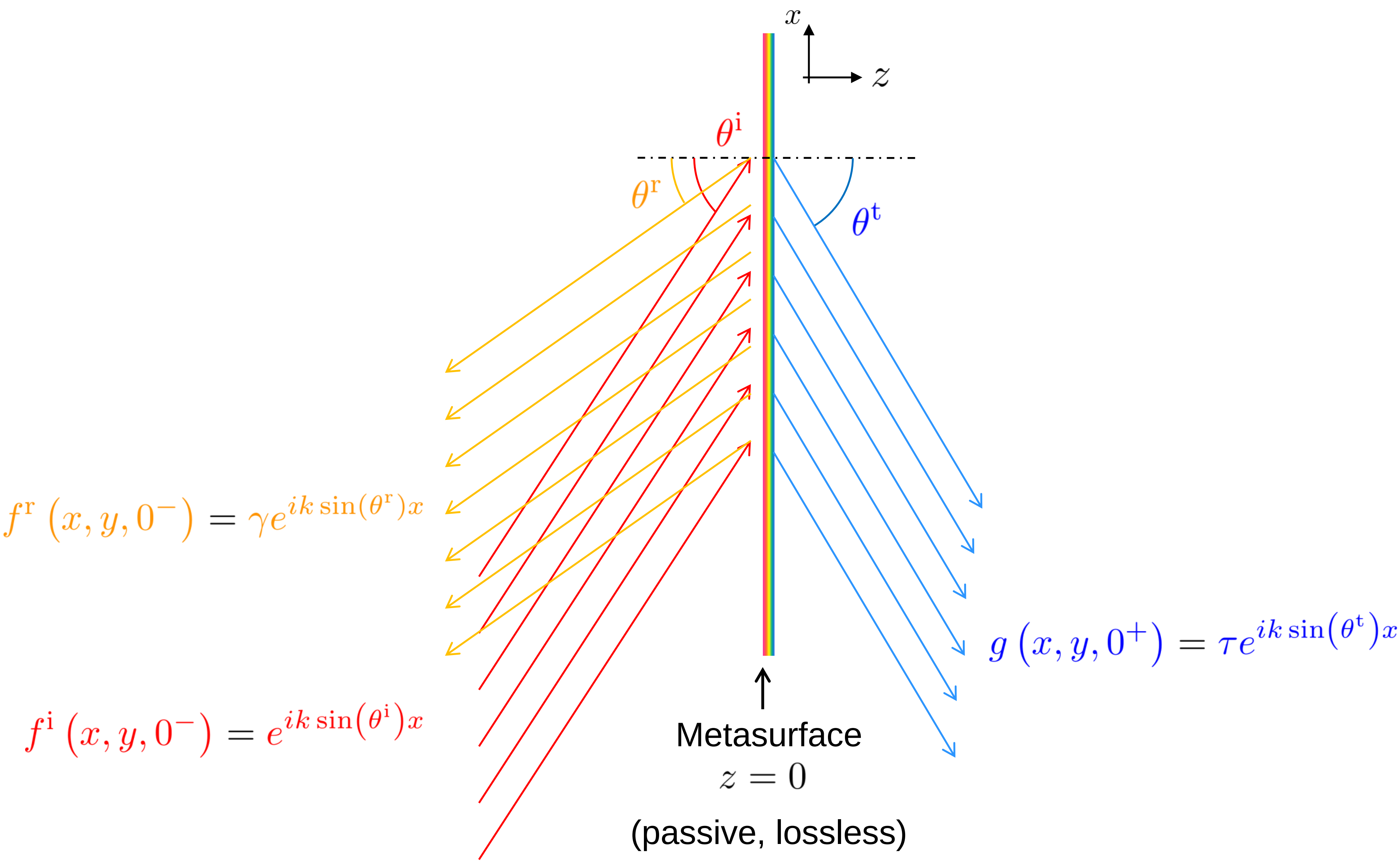
Complex amplitude transmittance (CAT):

$$t\left(x, y\right) = \frac{g\left(x, y, 0^+\right)}{f^{\mathrm{i}}\left(x, y, 0^-\right)}$$

Definition of input:

$$f\left(x, y, 0^-\right) = f^{\mathrm{i}}\left(x, y, 0^-\right) + f^{\mathrm{r}}\left(x, y, 0^-\right)$$

Illustrative Example: Anomalous Reflection & Refraction



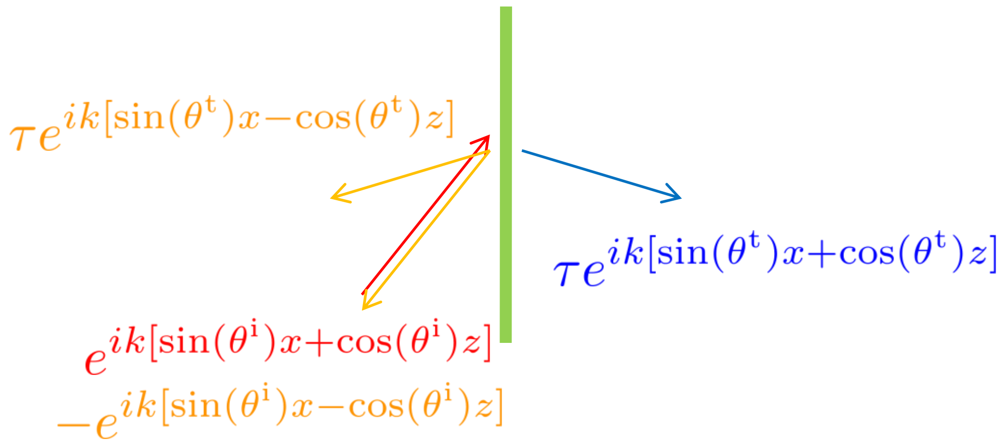
e.g. 2D problem, TE_y wave

CAT & Boundary Conditions

CAT Approach

$$t(x,y) = \frac{g(x,y,0^{+})}{f^{\mathrm{i}}(x,y,0^{-})} = \frac{\tau e^{ik\sin(\theta^{\mathrm{t}})x}}{e^{ik\sin(\theta^{\mathrm{i}})x}} = \tau \exp(ikx[\sin(\theta^{\mathrm{t}}) - \sin(\theta^{\mathrm{i}})])$$

What about the reflected wave?



Cannot be used for synthesis:

Not the required behavior

Violating passivity

Boundary conditions

(2D-TE)

$$f(x,y,0^{-}) = g(x,y,0^{+})$$

$$\partial_z\, f(x,y,z)\big|_{z=0^{-}} = \partial_z\, g(x,y,z)\big|_{z=0^{+}}$$

$$f(x,y,z) = f^{\mathrm{i}}(x,y,z) + f^{\mathrm{r}}(x,y,z)$$

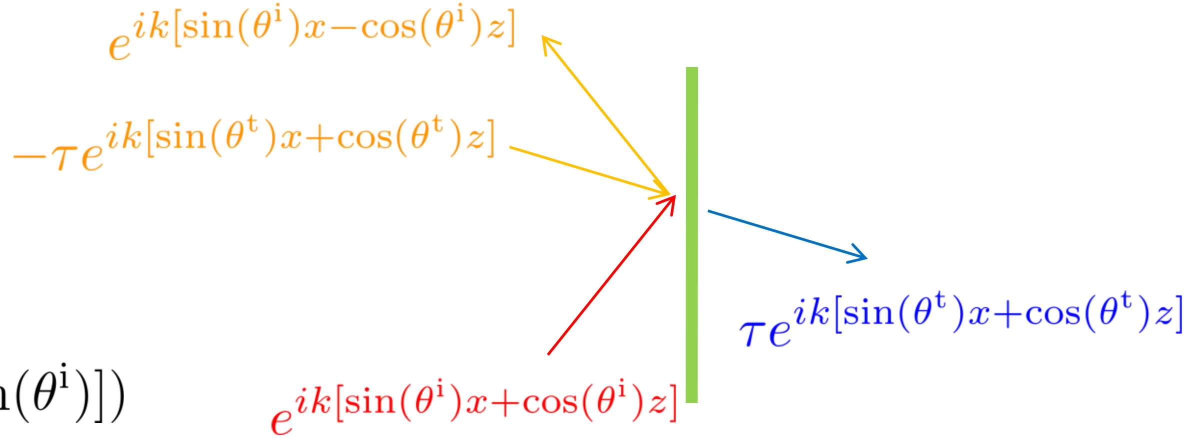
$$f^{\mathrm{r}}(x,y) = g(x,y) - f^{\mathrm{i}}(x,y) = \tau e^{ik\sin(\theta^{\mathrm{t}})x} - e^{ik\sin(\theta^{\mathrm{i}})x}$$

$$f^{\mathrm{r}}(x,y,z) = \tau e^{ik[\sin(\theta^{\mathrm{t}})x-\cos(\theta^{\mathrm{t}})z]} - e^{ik[\sin(\theta^{\mathrm{i}})x-\cos(\theta^{\mathrm{i}})z}$$

CAT & Energy Conservation

CAT Approach

$$t(x,y) = \frac{g(x,y,0^+)}{f^{\mathrm{i}}(x,y,0^-)} = \frac{\tau e^{ik\sin(\theta^{\mathrm{t}})x}}{e^{ik\sin(\theta^{\mathrm{i}})x}} = \tau\exp(ikx[\sin(\theta^{\mathrm{t}})-\sin(\theta^{\mathrm{i}})])$$



Energy Conservation

(same medium)

$$f^{\mathrm{i}}(x,y,0^-) = f^{\mathrm{r}}(x,y,0^-) + g(x,y,0^+) = f^{\mathrm{r}}(x,y,0^-) + t(x,y)f^{\mathrm{i}}(x,y,0^-)$$

$$f^{\mathrm{r}}(x,y,0^-) = (1-t(x,y))\,f^{\mathrm{i}}(x,y,0^-)$$

Cannot be used for synthesis:

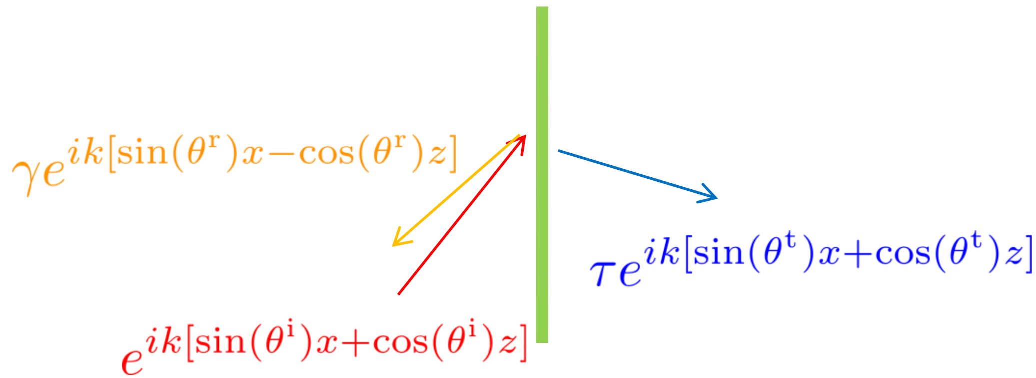
Not the required behavior

Violating causality

MT & Boundary Conditions

MT Approach

$$\eta\left(x,y\right) = \frac{g\left(x,y,0^+\right)}{f\left(x,y,0^-\right)} = \frac{\tau e^{ik\sin(\theta^{\mathrm{t}})x}}{e^{ik\sin(\theta^{\mathrm{i}})x} + \gamma e^{ik\sin(\theta^{\mathrm{r}})x}}$$



Boundary conditions

(in the sense of distributions)

$$g(x,y,0^+) - f(x,y,0^-) = \Lambda(x,y)$$

$$\partial_z\, g(x,y,z)|_{z=0^+} - \partial_z\, f(x,y,z)|_{z=0^-} = \underbrace{\Lambda'(x,y)}$$

Can be used for synthesis:
Correct physical description

Related to $\eta\left(x,y\right)$

$$\Lambda\left(x,y\right) = \left(\eta\left(x,y\right)-1\right) f\left(x,y,0^-\right)$$

$$f^{\mathrm{r}}(x,y,0^-) = g(x,y,0^+) - f^{\mathrm{i}}(x,y,0^-) - \Lambda(x,y) = \gamma e^{ik\sin(\theta^{\mathrm{r}})x}$$

MT & Boundary Conditions, Apparent Paradox

Boundary conditions

(in the sense of distributions)

$$g(x,y,0^+) - f(x,y,0^-) = \Lambda(x,y)$$

Momentum conservation (physics)

$$\tilde{g}(k_x,k_y;0^+) - \tilde{f}(k_x,k_y;0^-) = \tilde{\Lambda}(k_x,k_y) \quad \Longleftrightarrow \quad \tilde{g}(k_x,k_y;0^+) - \tilde{f}(k_x,k_y;0^-) = 0$$

~~Paradox?~~

Discontinuous momenta

(require distribution notation)

$$\langle \tilde{g}, \tilde{\phi} \rangle - \langle \tilde{f}, \tilde{\phi} \rangle = \langle \tilde{\Lambda}, \tilde{\phi} \rangle$$

$$\langle \tilde{\Lambda}, \tilde{\phi} \rangle = \langle 0, \tilde{\phi} \rangle$$

Particular solution: $\tilde{\Lambda} = 0$

Homogeneous solution: $\operatorname{supp}\left[\tilde{\Lambda}\right] = 0 \quad \Longrightarrow \quad \tilde{\Lambda} = \Lambda(x,y)\delta(k_x,k_y)$

From momentum transformation: $\Lambda = (\eta - 1)\, f$

(Surface response function)

M. A. Salem and C. Caloz, Opt. Express, 22, 2014.

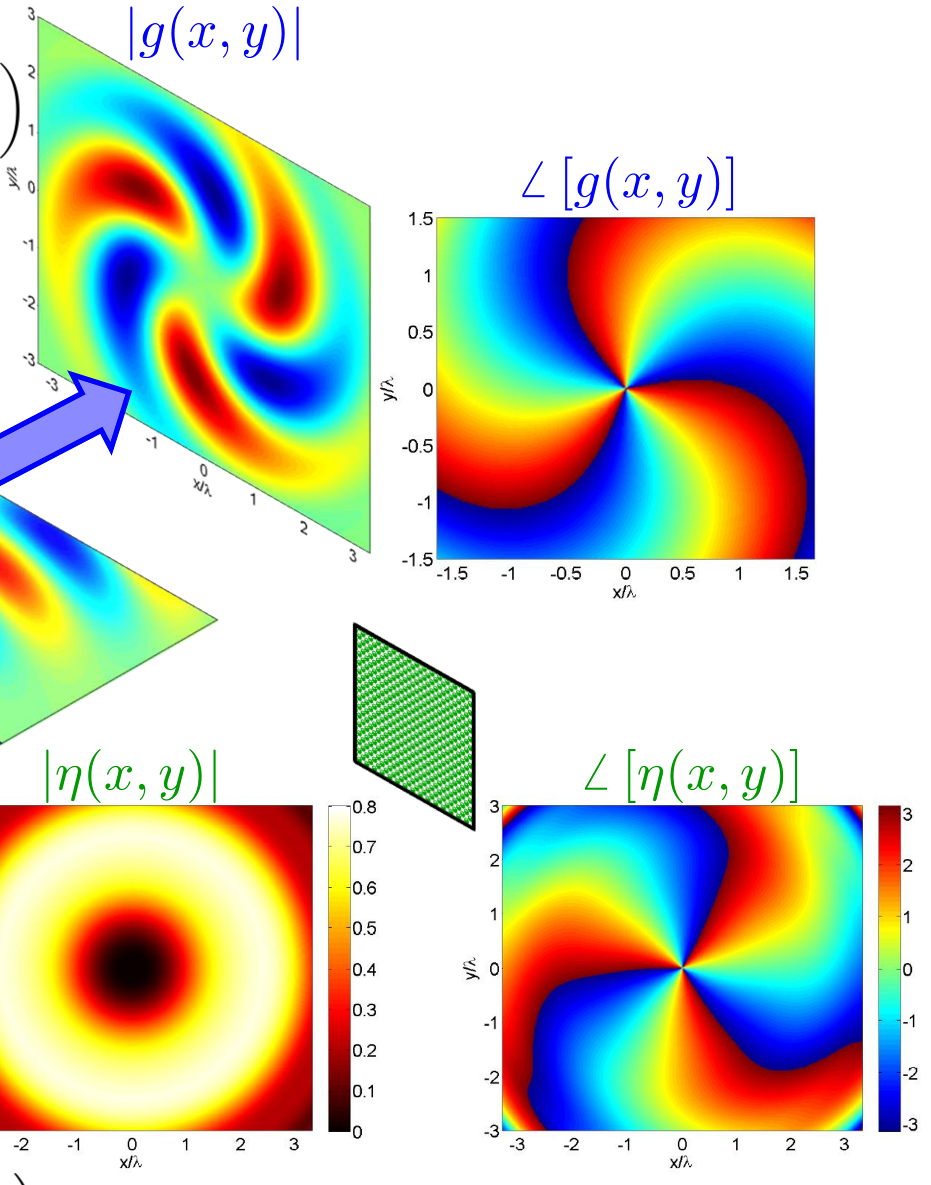
Straigthforward generalization to vector case !

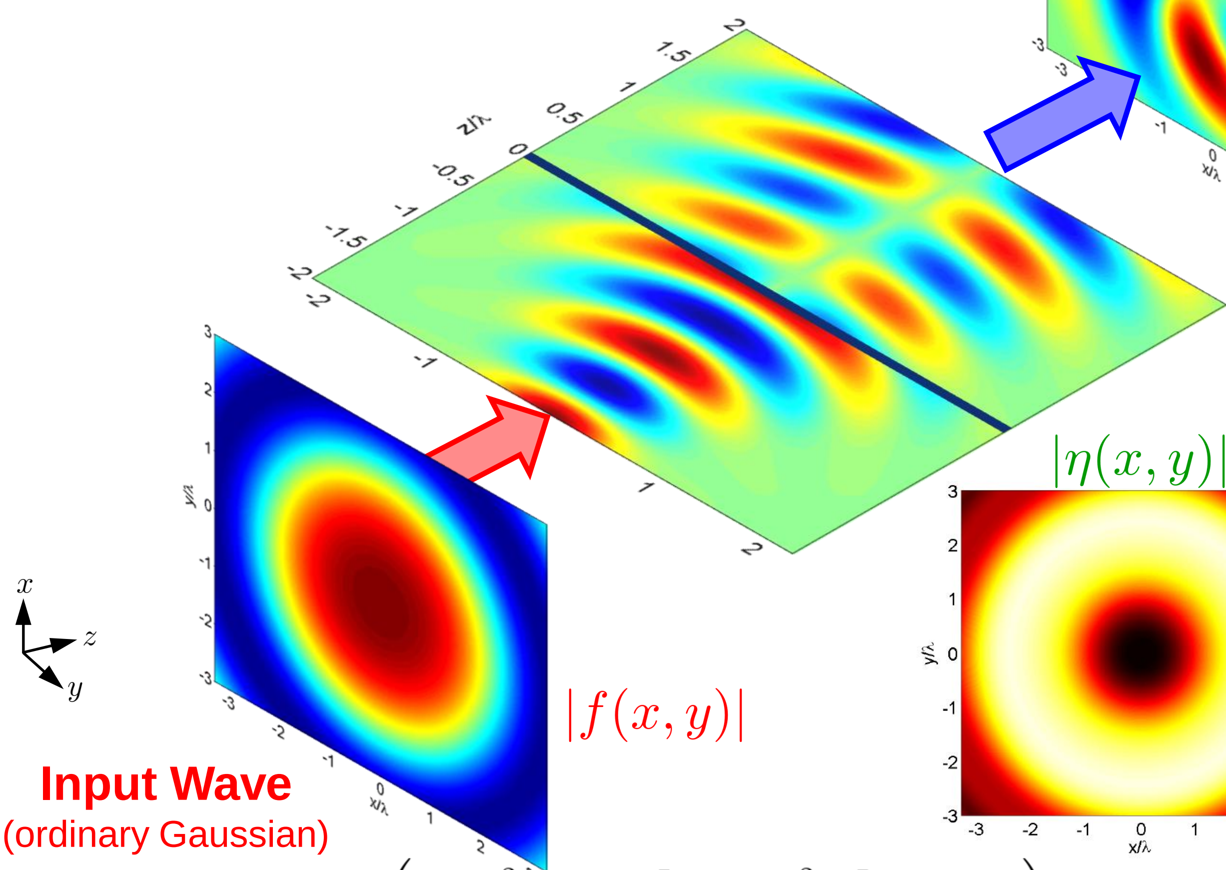
Outline

Stable Vortex Beam Generation

Output Wave
(vortex hypergeometric Gaussian)

$$g\left(x,y,z\right)=\frac{i^{|m|+1}\xi^{p/2}}{\left[\xi+i\right]^{1+|m|+p/2}}\left[\frac{\rho}{w_0}\right]^{|m|}\exp\left(\frac{-i\rho^2}{w_0^2\left[\xi+i\right]}+im\phi\right)$$
$$\times\frac{\Gamma\left(1+|m|+\frac{p}{2}\right)}{\Gamma\left(|m|+1\right)}\,{}_1F_1\left(-\frac{p}{2},|m|+1;\frac{\rho^2}{w_0^2\xi\left[\xi-i\right]}\right)$$

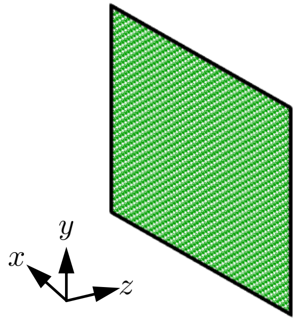




Input Wave
(ordinary Gaussian)

$$f\left(x,y,z\right)=\frac{w_0}{w\left(z\right)}\exp\left(-\frac{r^2}{w\left(z\right)^2}-ik\left[z+\frac{r^2}{2R\left(z\right)}\right]-i\vartheta\left(z\right)\right)$$

Delay-Start Airy Beam Generation



Input Wave
(normally incident plane wave)

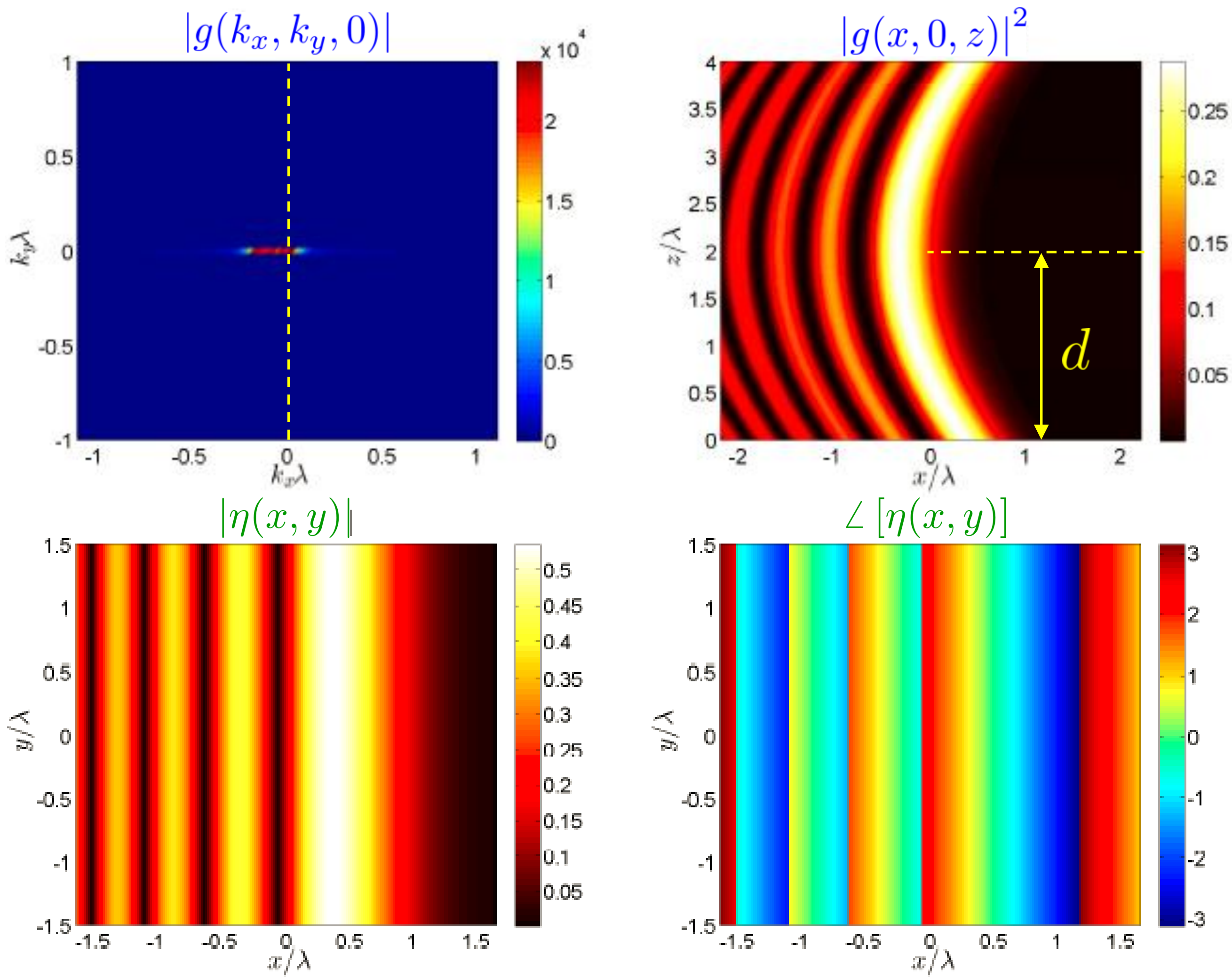
$$f(z) = e^{ikz}$$

Output Wave (Airy beam with start at z = d)

$$g\left(x, z\right) = \mathrm{Ai}\left(s - \frac{\zeta^2}{4}\right)\exp\left(i\left[\frac{s\zeta}{2} - \frac{\zeta^3}{12}\right]\right)$$

$$\tilde{\varphi}(k_x; k, d) = \exp\left(-i\sqrt{k^2 - k_x^2}\,d\right)$$

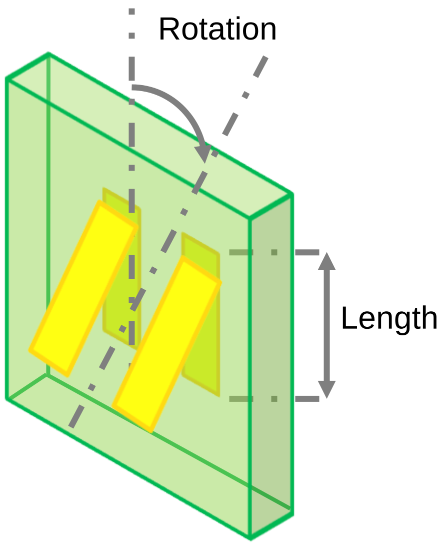
$$s = x/x_0,\ \zeta = z/kx_0^2$$



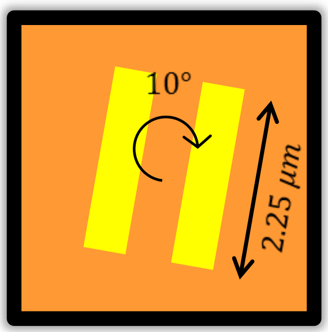
Implementation via Parametric Mapping

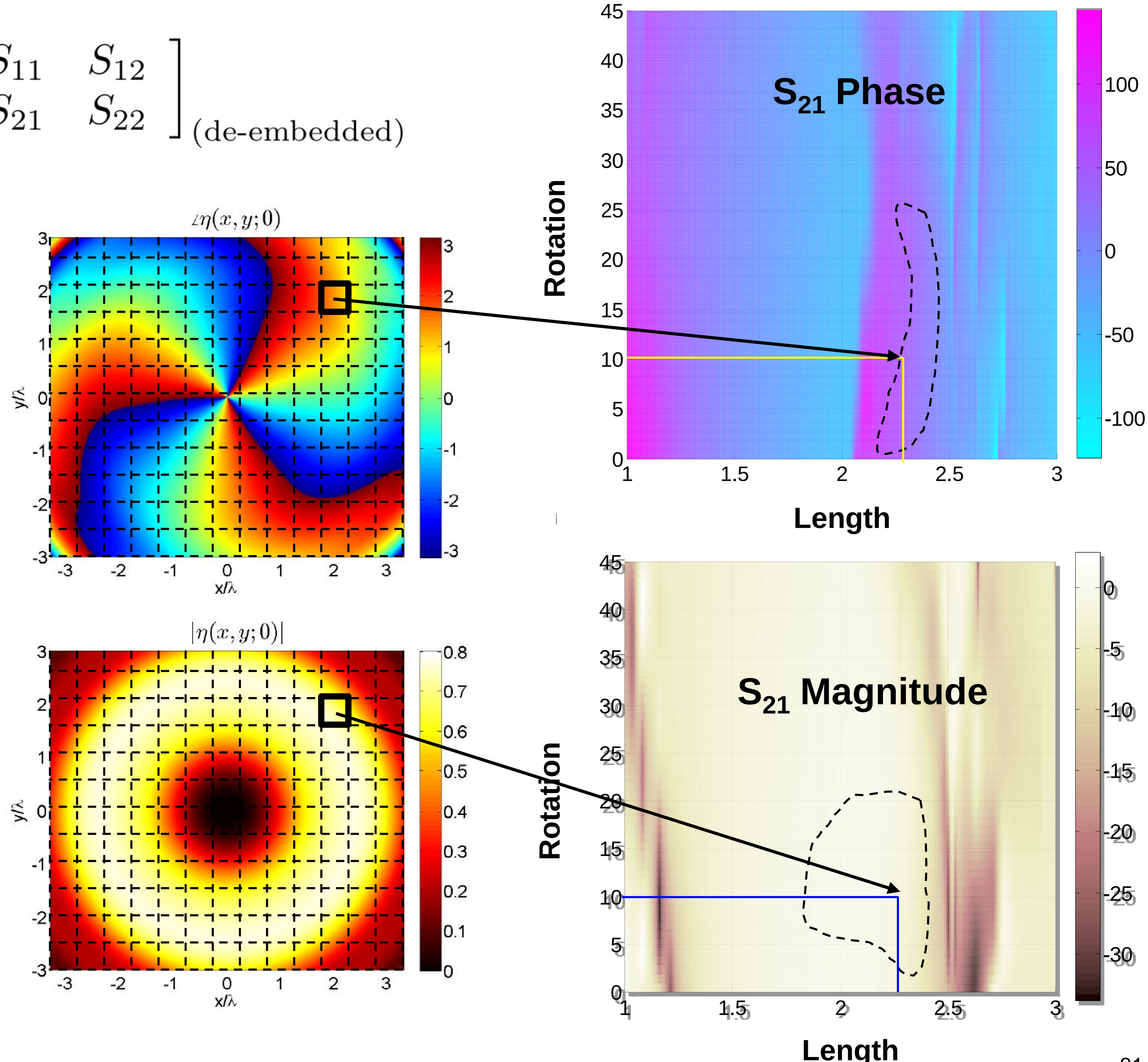
$$\eta(x,y) \Longrightarrow \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\text{(de-embedded)}}$$

Scatterer topology



Pixel Dimensions





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II. MT Method

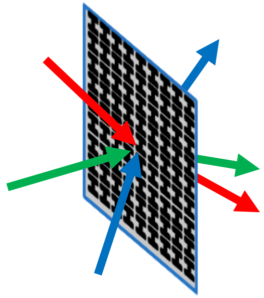
III. Comparison with Optics

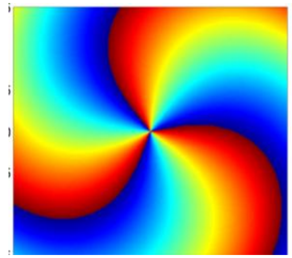
IV. Implementation Suggestion

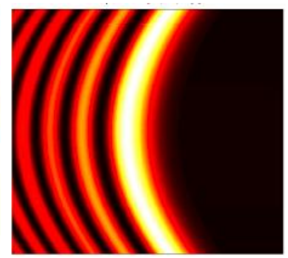
❖ Conclusions & Questions

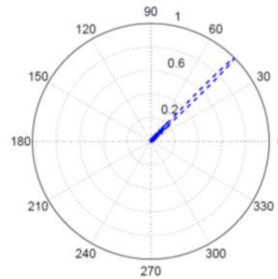
Conclusions

- **Metasurfaces:** unprecedented EM control

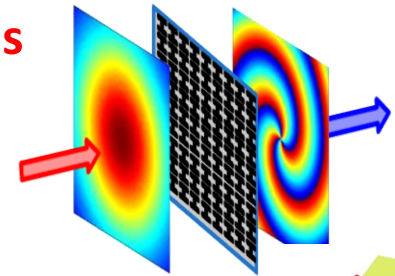




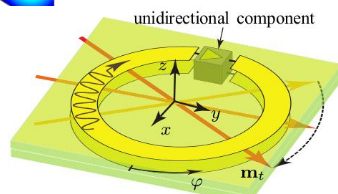


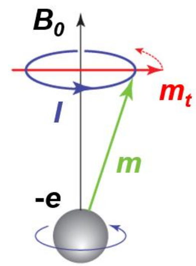


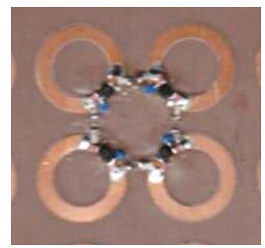
- **No universal synthesis** – until now



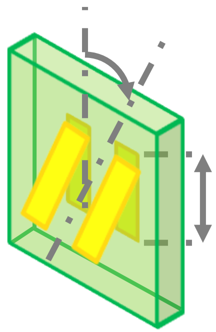
- **Synthesis based on transformation in momentum space for physical insight**

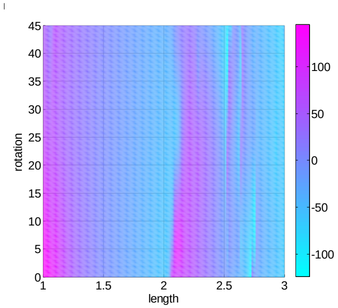


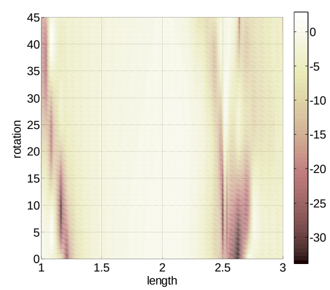




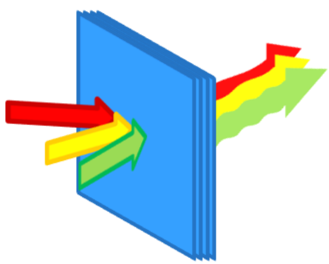
- **Implementation using lookup maps**







- **Potential for myriads of applications**



K. Achouri, M. A. Salem and C. Caloz, "General Metasurface Synthesis Based on Susceptibility Tensors," arXiv:1408.0273.